Ministero dell'Istruzione, dell'Università, della Ricerca
Ufficio Scolastico Regionale Per Il Lazio
Liceo Scientifico Statale
"TALETE"

Modulo 3 MiniCLIL

Incontro 10: Sequences and Series/3

L'UdA odierna si articola su 3 ore di intervento che permetterà la proiezione e l'analisi ragionata del film in lingua originale *'The Man Who Knew Infinity'*. Il film di Michael Brown crea due linee narrative ponendo l'accento sulle due relazioni: quella tra Ramanujan e la figura del suo maestro e matematico G.H. Hardy e quella tra Ramanujan e l'Inghilterra, in un continuo confronto tra i due mondi in cui l'universalità dei principi matematici sanno elevarsi al di sopra di pregiudizi e differenze linguistiche.

Lezione

La proiezione del Film e lo svolgimento dell'incontro avverrà in sala video dell'Istituto Talete mediante utilizzo dell'ambiente Netflix. Il commento e l'analisi dei contenuti filmici sarà inoltre guidato da dispense cartacee distribuite in fotocopia e da un'introduzione del Prof. Béla Bollobás, qui in allegato e in link.
`https://www.youtube.com/watch?v=fGFK7rhpbWk`

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling'.

Just as in elementary mathematics you give a meaning to when is negative and fractional to conform to the law which holds when is a positive integer, similarly the whole of my investigations proceed on giving a meaning to Eulerian Second Integral for all values of . My friends who have gone through the regular course of University education tell me that is true only when is positive. They say that this integral relation is not true when   is negative. Supposing this is true only for positive values of and also supposing the definition to be universally true, I have given meanings to these integrals and under the conditions I state the integral is true for all values of negative and fractional. My whole investigations are based upon this and I have been developing this to a remarkable extent so much so that the local mathematicians are not able to understand me in my higher flights.

Very recently I came across a tract published by you styled Orders of Infinity in page 36 of which I find a statement. that no definite expression has been as yet found for the number of prime numbers less than any given number. I have found an expression which very nearly approximates to the real result, the error being negligible. I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

I remain, Dear Sir, Yours truly,
S. Ramanujan

The Man Who Knew Infinity: inspiration, rigour and the art of mathematics

The movie *The Man Who Knew Infinity* is about Srinivasa Ramanujan, who is generally viewed by mathematicians as one of the two most romantic figures in our discipline.

Ramanujan (1887–1920) was born and died, aged just 32, in Southern India. But in one of the most extraordinary events in mathematical history, he spent the period of World War I in Trinity College Cambridge at the invitation of the leading British mathematician Godfrey Harold (G. H.) Hardy (1877–1947) and his great collaborator John E. Littlewood.

Suffice to say that as a boy he refused to learn anything but mathematics, he was almost entirely self taught and his pre-Cambridge work is contained in a series of Notebooks.

The work he did after returning to India in 1919 is contained in the misleadingly named Lost Notebook. It was lost and later found in the Wren library of the leading college for mathematics of the leading University in England. While in England Ramanujan became the first Indian Fellow both of Trinity and of the Royal Society.

A Man Of Numbers

Ramanujan had an extraordinary ability to see patterns. While he rarely proved his results he left a host of evaluations of sums and integrals. He was especially expert in a part of number theory called modular forms which is of even more interest today than when he died.

The lost notebook initiated the study of mock theta functions which are only now being fully understood. Fleshing out his Notebooks has only recently been completed principally by American mathematicians Prof Bruce Berndt and Prof George Andrews. It comprises thousands of printed pages.

An old Indian friend, Swami Swaminathan, oversaw the Ramanujan Library in Madras over half a century ago. He commented that had Ramanujan been born ten years early he would have been unable to receive the education and financial assistance that made his pre-Cambridge work possible.

Swaminathan went on to say that had Ramanujan been born ten years later, he would have probably received a more robust and more ordinary education. In either case our version of Ramanujan would not exist.

The Man Who Knew Infinity presents some of the best descriptions of mathematics yet seen in a film.

Based on a book of the same name by Robert Kanigel, it is the story of the mathematician Srinivasa Ramanujan (played by Dev Patel) from Tamil Nadu in India.

Initially, Ramanujan works alone in Madras, India, self-taught from books but taking his mathematics beyond the frontiers of the field, attributing his inspiration to the Hindu goddess Namagiri.

He writes a long letter full of equations to the brilliant Cambridge mathematician G.H. Hardy (Jeremy Irons), who is so impressed by Ramanujan's remarkable discoveries that he brings him to Trinity College, Cambridge, in 1914.

Ramanujan leaves his wife and mother behind, and does not return to India until 1919.

In Cambridge, Hardy and Ramanujan collaborate, with the first world war as backdrop. But there is considerable tension between their approaches: the untutored genius Ramanujan, with his intuition and inspiration; the dry, crusty Hardy, traditionally educated and steeped in the formality and rigour of the professional mathematician.

Hardy is joined at times by his Trinity colleague and close collaborator John Littlewood (Toby Jones).

Ramanujan is anxious to see his discoveries published, but experiences frustration due to Hardy's insistence on rigorous proof.

Littlewood raises doubts about parts of Ramanujan's work, and some of his results are found to be incorrect.

Ramanujan must learn to supplement his inspired explorations with technical discipline, so that his discoveries can be verified to be true and his efforts not wasted.

Hardy, in turn, is a strenuous advocate for him, eventually leading to the highest levels of recognition by British academia.

The nature of mathematics
I expect this film will lead to better-informed public perceptions of mathematics.

Too often, the field is seen as a set of methods for manipulating numbers, shapes or symbols. These methods are important and useful, but they are products of mathematics and tools used by it; they are not the essence of the field.

The film presents mathematics as art and as a creative process of discovery.

To Ramanujan, it reveals “thoughts of God”. Hardy, an atheist, would not describe it in such terms, but has a similar aesthetic sense.

Most mathematicians would agree with Hardy that mathematical facts (theorems) are discovered rather than invented. Hardy once described them as “notes of our observations”. They exist as fundamental truths independent of the activity of human beings, independent of time itself: their truth predates our recognition of them.

The film captures much of the spirit of mathematical research. The mathematician is guided by curiosity and seeks beautiful and elegant connections between abstract concepts.

These explorations typically involve some experimentation, but with ideas and symbols rather than physical things.

There are usually lots of mistakes and dead ends. Great persistence is needed. If a new connection is believed to have been found, there is a unique intellectual thrill and maybe a sense of awe.

But the game is not over. A proof – a complete, verifiable, logical justification – must be built. Constructing the proof can be difficult and often takes a lot longer than the initial discovery.

It is tempting to move on to try to discover other connections, rather than work on the proofs to support the ones already found, but we must learn to persist until the proof is finished. Tertiary education in mathematics aims to instil this.

In India, Ramanujan lacked such an education. At Cambridge, he had to catch up and fill in gaps.

You can read more about his time in Cambridge, and Hardy’s mentoring, in an article by Bela Bollobas.

Students, teachers and practitioners of mathematics will see some good practices in this film.

Here is one. In Madras, paper is expensive and Ramanujan must constrain his use of it. But at Cambridge he is empowered by the large pile of blank sheets he finds on his desk.

Having plenty of paper available for working out – even if the answer has to fit within some box on a worksheet or an exam paper – allows the mind full flight.

Primes and partitions

The Man Who Knew Infinity touches on topics that at the time lay at the forefront of pure mathematics but today are connected to a multitude of practical applications. Some of Ramanujan's early work is on prime numbers: a number is prime if no other numbers divide it exactly except for itself and one.

These are the "atoms" from which every number is made; they are "bonded" together by multiplication. Mathematicians have long been interested in how the prime numbers are distributed among all whole numbers, and in the intriguing patterns they form.

We have good estimates for how often prime numbers tend to occur as we travel along the number line. The formal statement, known as the Prime Number Theorem, was proved in 1896.

Early in his correspondence with Hardy, Ramanujan proposed a more precise version of the theorem. Alas, this version was wrong, a serious disappointment at the time. Nonetheless Ramanujan went on to do important work with primes.

In those days, prime numbers were of interest only for their beauty and their curious behaviour. Hardy himself, at heart the purest of pure mathematicians, rejoiced in the fact that his field had no practical applications. But today prime numbers underpin the cryptographic algorithms used to provide secrecy and authentication in most electronic transactions and communications.

The other mathematical topic that features in the film is partitions, in which a positive integer is expressed as a sum of other positive integers.

In how many ways can this be done? The film illustrates this with the five partitions of 4, namely, 1+1+1+1, 2+1+1, 2+2, 3+1, and 4 itself.

The count of partitions increases as the positive integer itself gets larger – there are 204,226 partitions of 50 – but in a way that is difficult to capture with a simple formula.

The expert on the topic was Major Percy MacMahon, who used the long-winded methods of the day to work out the partition count for all numbers up to 200.

Hardy and Ramanujan developed a completely new approach, using mathematical ingredients that seem, at first sight, out of place when working with positive whole numbers: integral calculus, which concerns areas bounded by curves, complex numbers, involving the square root of -1, and elliptic functions, which generalise the circular functions – sin, cos and tan etc – that many of us met at school.

They could then calculate numbers of partitions much more efficiently, to the amazement of their peers. Partitions are part of combinatorics, the mathematics of arranging objects in patterns according to rules. Combinatorics was then seen as young and recreational, but now permeates the modern world through the design and analysis of network structures, encoding schemes and optimisation algorithms.